SLR-VA – 1

Seat No.	

M.Sc. – I (Semester – I) Examination, 2014
MATHEMATICS (Paper – I)
Object Oriented Programming Using C++

Day and Date : Monday, 21-4-2014 Max. Marks : 70
Time : 11.00 a.m. to 2.00 p.m.

Instructions : *1) Question No. **1** and **2** are **compulsory**.*
*2) Attempt **any three** questions from Q. No. **3** to Q. No. **7**.*
*3) Figures to the **right** indicate **full** marks.*

1. A) Choose correct alternatives : **10**

i) Which of the following is not a type of constructor ?
a) Copy constructor b) Friend constructor
c) Default constructor d) Parameterized constructor

ii) A ________________ is a collection of objects of similar type.
a) Class b) Object
c) Inheritance d) None of the above

iii) The process of making an operator to exhibit different behaviours in different instances is known as ________________
a) Function overloading b) Inheritance
c) Operator overloading d) None of the above

iv) The smallest individual unit in a program are known as _____________
a) Tokens b) Object
c) Class d) None of the above

v) Which of the following is not the member of class ?
a) Static function b) Friend function object
c) Const function d) None of the above

vi) Exceptions are _________________
a) Logical error b) Compiler error
c) Runtime error d) Syntactic error

P.T.O.

vii) Graphical representation of a problem is known as _______________

a) Algorithm b) Flowchart

c) Program d) None of the above

viii) ___________ are operators that are used to format the data display.

a) Manipulators b) Object

c) Inheritance d) None of the above

ix) Which of the following concepts means wrapping up of data and functions together ?

a) Abstraction b) Encapsulation

c) Inheritance d) Polymorphism

x) Conditional operators are also known as ___________

a) Ternary operator b) Relational

c) Assignment operator d) Arithmetic operator

B) State whether the following statements are **true** or **false** : **4**

i) In C++, declarations can appear almost anywhere in the body of a function.

ii) A C++ function can return multiple values to the calling function.

iii) A non member function may have access to the private data of a class if it is declared as a friend of that class.

iv) Constructors cannot be virtual.

2. A) Write short notes on the following : **8**

i) Function prototyping

ii) Call by reference and return by reference.

B) Answer the following : **6**

i) Define algorithm. Explain characteristics of algorithm.

ii) Explain inline function with suitable example.

3. Answer the following : **14**

   A) What is constructor ? Explain Parameterized constructor with example.

   B) Write a C++ program to study the use of Friend function.

4. Answer the following : **14**

   A) What is template ? Explain function template with suitable example.

   B) Write a C++ program to study the use multiple inheritance (assume own data).

5. Answer the following : **14**

   A) What is function overloading ? Explain with suitable example.

   B) Explain virtual function with suitable example.

6. Answer the following : **14**

   A) Explain following function with suitable example.

      a) put() and get() functions.

   B) Write a C++ program to print the square of even numbers from 0 to 100.

7. Answer the following : **14**

   A) What is file ? Explain the various functions involved in opening and closing a file.

   B) Explain the difference between structure and class in detail.

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SLR-VA – 10

Seat No.	

M.Sc. – I (Semester – II) Examination, 2014
MATHEMATICS
Relativistic Mechanics (Paper No. – X)

Day and Date : Friday, 2-5-2014 Max. Marks : 70
Time : 11.00 a.m. to 2.00 p.m.

Instructions : *1) Q. No.* ***1*** *and Q. No.* ***2*** *are* ***compulsory****.*
2) Attempt ***any three*** *questions from Q. No.* ***3*** *to Q.No.* ***7****.*
3) Figures to the ***right*** *indicate* ***full*** *marks.*

1. A) Fill in the blanks (**one** mark **each**) : **5**

1) A region where in a small magnet or a loop of wire carrying current will experience a force is called __________

2) The total charge in an isolated system is ________ by the motion of its charge carriers.

3) For low values of velocity V, the Lorentz transformation approaches to ____________

4) An unaccelerated frame is __________ frame.

5) In the process of contraction the rank of tensor is reduced by ______.

B) Choose the correct alternative (**one** mark **each**) : **5**

1) The fourth component of four momentum is _______________

a) mass b) velocity of light
c) momentum d) energy

2) The comparison between the _______ of wave motion as observed by the observers in s and s^1 frames is called the Doppler effect .

a) Direction of propagation of wave
b) Measures of frequency
c) Measures of wavelength
d) Measures of velocities

P.T.O.

3) Consider the constant ratio $\frac{v_1 - v_2}{u_1 - u_2} = -e$

Then the collision is elastic if ______

a) e = 0 b) e = 1 c) e = – 1 d) e > 0

4) The interval between two events in Minkowski's space is _______ of frame of reference.

a) independent b) dependent

c) simultaneous d) co-incident

5) If the charges are at rest then the current density is ______

a) finite b) infinite c) zero d) one unit

C) State **true** or **false** (**one** mark **each**) : **4**

1) Accelerated Frames are non-inertial.

2) The length of a rod remains unchanged in a direction perpendicular to the direction of motion.

3) The composition of two velocities which are separately less than c, will always exceed the velocity of light.

4) The time recorded by a clock moving with a given system is called proper time for that system.

2. a) The length of a rocket ship is 100 metres on the ground. When it is in flight its length observed on the ground is 99 metres. Calculate its speed. **3**

b) Give the formula for Einstein's addition of two velocities u' and v. State the resultant when the two velocities u' and v are comparable with the velocity of light. **3**

c) Prove that : Kronecker delta symbol is a mixed tensor of rank 2. **4**

d) Write the formula for relativistic expression of mass and show that if u <<< c then $m = m_0$. **4**

3. a) Derive the transformation equations for electric field. **8**

b) Show that : Newton's laws of motion are invariant under Galilean transformations. **6**

4. a) Prove or Disprove : Electromagnetic wave equation is invariant under Galilean transformation. **7**

b) In a inelastic collision, a particle of rest mass m_0 and kinetic energy $2m_0c^2$ strikes and sticks to a stationary particle of rest mass $2m_0$. Calculate the rest mass of the composite particle. **7**

5. a) Derive the transformation rules for momentum and energy of a particle. **7**

b) Show that : Minkowski's space time interval between two events is an invariant quantity under Lorentz transformation. **7**

6. a) Prove that : The following Maxwell's equations are invariant under Lorentz transformation.

$$\text{div } \bar{B} = 0; \text{ curl } \bar{E} = \frac{-\partial \bar{B}}{\partial t}.$$ **10**

b) Calculate the velocity at which the mass of particle becomes 8 times its rest mass. **4**

7. a) With usual notations show that, $\dfrac{c^2 - u'^2}{c^2} = \dfrac{(c^2 - u^2)(c^2 - v^2)}{(c^2 - ux^v)^2}$ **10**

b) Write a note on :

i) Inner product of two tensor

ii) Outer product of two tensor. **4**

Seat No.	

M.Sc. (Part – II) (Semester – III) Examination, 2014
MATHEMATICS
Functional Analysis (Paper – XI)

Day and Date : Monday, 21-4-2014 Max. Marks : 70
Time : 3.00 p.m. to 6.00 p.m.

N.B. : *1) Q. **1** and Q. **2** are **compulsory**.*
*2) Attempt **any three** questions from Q. **3** to Q. **7**.*
*3) Figures to the **right** indicates **full** marks.*

1. A) Fill in the blanks (**one** mark **each**) : **5**

1) The Schwarz inequality is given by $|\langle x, y\rangle| \leq$ ___________

2) Any $x \in H$ is such that $\|x\| = 1$ then x is said to be ___________

3) If A_1, A_2 are self adjoint on H then their product $A_1.A_2$ is self adjoint if and only if ___________

4) A linear transformation T on a linear space V is a projection on some subspace iff T is an ___________ operator.

5) If $T \to T^*$ is an adjoint operation then for any scalar 'α', $(\alpha t)^* =$ ___________

B) State whether following statements are **true** or **false** (**One** mark **each**) : **4**

1) An orthonormal set contains a zero vector.

2) Every unitary operator is a normal operator.

3) For any bounded linear operator $T : X \to Y$, the null space N(T) is a closed subspace of X.

4) Every convergent sequence in a normed linear space X is Cauchy.

C) Choose the correct alternative (**One** mark **each**) : **5**

1) For any unitary operator T, $\|T\| =$ ___________

a) 1 b) 0 c) –1 d) $\sqrt{2}$

2) An operator T on a complex Hilbert space H is self adjoint iff $\langle Tx, x\rangle$ is ___________ for all x.

a) zero b) non-zero

c) real d) complex

3) If S_1 and S_2 are any two non-empty subsets of H then $S_1 \subset S_2 \Rightarrow$ ________

a) $S_1^{\perp} \subset S_2^{\perp}$ b) $S_2^{\perp} \subset S_1^{\perp}$

c) $S_2^{\perp\perp} \subset S_1^{\perp\perp}$ d) All of these

4) Which of the following statement is false ?

a) $x \perp y \Rightarrow y \perp x$ b) $x \perp y \Rightarrow \alpha x \perp y$

c) $x \perp x \Rightarrow x = 0$ d) None of these

5) If Y is complete space then B (X, Y) is ________

a) Complete b) Not complete

c) Compact d) Totally bounded

2. a) Let $T : X \to Y$ be defined by T(x) = 0 for all $x \in X$ then prove that T is a bounded operator and $\|T\| = 0$. **4**

b) Let X be a normed linear space and $S, T : X \to X$ bounded linear operators and (ST) (x) = S [T(x)] then prove that ST is a linear operator. **3**

c) Let X be a complex inner product space then prove that,
$\langle x, dy + \beta z\rangle = \overline{\alpha}\,\langle x, y\rangle + \overline{\beta}\langle x, z\rangle$. **4**

d) If x, y are any two vectors in a Hilbert space then prove that,
$\|x + y\|^2 - \|x - y\|^2 = 4$. Real $\langle x, y\rangle$. **3**

3. a) If S is a non-empty subset of a Hilbert space H then prove that $S^{\perp}$ is a closed linear subspace of H and hence Hilbert space. **7**

b) If M is a closed linear subspace of a Hilbert space H then prove that
$H = M \oplus M^{\perp}$ **7**

4. a) If H is a Hilbert space then prove that H^* is also Hilbert space with the inner product defined by, $\langle f_x, f_y \rangle = \langle y,\ x \rangle$. **8**

   b) The mapping $T \rightarrow T^*$ is an isometric isomorphism of $B(N) \rightarrow B(N^*)$ then prove that it reverses the product preserved identity transformation. **6**

5. a) Let S(x, r) be an open sphere in B with centre at x and radius r. Sr is the open sphere with centre at origin and radius r then prove that,

   i) S(x, r) = x + s (o, r)

   ii) S(o, r) = r – s(o, 1) **6**

   b) Let X be a normed linear space and $S = \{x \in X / \|x\| \leq 1\} = [0,\ 1]$ be a subspace of X. Show that X is a Banach space if and only if S is a complete. **8**

6. a) Show that the real and complex linear space are Banach spaces under the norm, $\|x\| = |x|$; $x \in R$ or C. **7**

   b) Let N be a normed linear space over k ($\mathbb{R}$ or $\mathbb{C}$). Let x_0 be any non-zero vector in N. Then prove that there exist a functional F in N^* such that,

   $f(x_0) = \|x_0\|$ and $\| f \| = 1$. **7**

7. a) Let S be non-empty subset of a Hilbert space then prove that set of all linear combinations S of vectors in S is dense in H if and only if $S^{\perp} = \{0\}$. **6**

   b) State and prove Riesz-Representation theorem. **8**

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Seat No.	

M.Sc. (Part – II) (Semester – III) Examination, 2014
MATHEMATICS (Paper – XII)
Advanced Discrete Mathematics

Day and Date : Wednesday, 23-4-2014 Total Marks : 70
Time : 3.00 p.m. to 6.00 p.m.

Instructions : *1) Q. No.* ***1*** *and Q. No.* ***2*** *are* ***compulsory****.*
2) Attempt ***any three*** *questions from Q. No.* ***3*** *to Q. No.* ***7****.*
3) Figures to the ***right*** *indicate* ***full*** *marks.*

1. a) Choose correct alternative : **5**

i) Every finite lattice is _______________

a) Distributive b) Bounded
c) Non-distributive d) Complete

ii) If no elements succeds α then α in poset S is called ______________

a) Maximal element b) Minimal element
c) Constant element d) None of these

iii) A path which originates and ends in the same node is called ____________

a) Cycle b) Loop
c) Circuit d) All above

iv) Components of a forest are ____________

a) Trees b) Cyclic connected graph
c) Regular graph d) Simple Graph

v) If $A \subseteq S$ and | A | denotes number of objects then ____________

a) $|A| = |S| - |\overline{A}|$ b) $|\overline{A}| = |S| - |A|$
c) Both a) and b) true d) Both a) and b) false

P.T.O.

b) Fill in the blanks : **5**

i) A poset with lowest element zero is a complete lattice if ______________

ii) A complemented distributive lattice with 0 and 1 is ______________

iii) A disconnected graph contains ______________ number of connected graph.

iv) The geometric series of $\frac{1}{ax+1}$ is ___________

v) Km, n is regular graph if ______________

c) Define the term : **4**

i) Graph

ii) Tree

iii) Bipartite connected graph

iv) component of graph.

2. a) Show that there is no simple graph with 10 vertices and 46 edges. **3**

b) Prove that Boolean Ring is Commutative Ring. **3**

c) Draw all spanning trees of K_4 graph. **4**

d) If five points are chosen randomly in the interior of equilateral triangles with each sides of two units then show that at least one pair of points has separation less than one unit. **4**

3. a) If $(B,\wedge,\vee)$ be boolean algebra then prove that $(B, +, \cdot, 0, 1)$ is boolean ring. **7**

b) If L is lattice. For any a, b $\in$ L

$a \geq b$ such that

$a \vee c = b \vee c$

$a \wedge c = b \wedge c$

For some $c \in L$ then L is modular lattice iff a = b. **7**

4. a) If a and b be the elements of modular lattice then the lattice interval $I[a, a \vee b]$ and $I[a \wedge b, b]$ are isomorphic. **7**

   b) Let A be the adjacency matrix of a graph G with m vertices where m > 1 then show that ij^{th} entry of the matrix A^n gives the number of walks of length n from the vertex vi to the vertex vj. **7**

5. a) State and prove Bridge Theorem. **7**

   b) If G is connected graph with n vertices then show that following statements are equivalent.

      i) G is tree

      ii) G is acyclic graph

      iii) G is a connected graph with n-1 edges. **7**

6. a) Solve the recurrence relation

      $ar + 5ar–1 + 6\ ar– 2 = 3r^2 – 2r + 1$ with $a_0 = 1, a_1 = 2$. **7**

   b) Find the number of integer between 1 to 1000 both inclusive which are divisible by none of 2, 4, 8. **7**

7. a) Write note on matrix representation of graph with example. **7**

   b) G be a non-empty graph with at least two vertices then prove that G is bipartite graph if and only if it has no odd cycle. **7**

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SLR-VA – 13

Seat No.	

M.Sc. (Semester – III) Examination, 2014
MATHEMATICS (Elective – I)
Linear Algebra (Paper – XIII)

Day and Date : Friday, 25-4-2014 Total Marks : 70
Time : 3.00 p.m. to 6.00 p.m.

Instructions : *1) Figures in **right** indicate **full marks**.*
*2) Q. No.**1** and Q. No. **2** are **compulsory.***
*3) Solve **any three** questions from Q.No.**3** to Q.No.**7**.*

1. a) Select correct alternative. **5**

i) A form f on real or complex V(f) is called Hermitian Vector Space if ________ $\forall \alpha, \beta \in \nu$.

a) $f(\alpha,\beta) = f(\beta,\alpha)$ b) $f(\alpha;\beta) = -f(\beta,\alpha)$

c) $f(\alpha,\beta) = \overline{f(\beta,\alpha)}$ d) $f(\alpha,\beta) = 0$

ii) A complex n×n matrix A is called normal if __________

a) $A \cdot A^{\circ} = A^{\circ} \cdot A$ b) $A \cdot A^{*} = A^{*} \cdot A$

c) $A = A^{\circ}$ d) $A = A^{*}$

iii) Every finite dimensional inner product space has an ________ basis.

a) Orthogonal b) Orthonormal

c) Simple d) None of these

iv) Let W_1, W_2, -----,W_k be subspaces of V(f) we say that W_1, W_2, --- W_k are independent if __________

a) $\alpha_1 + \alpha_2 + ... + \alpha_k = 0, \alpha_i \in W_i$ and $\alpha_i = 0$

b) $\alpha_1 + \alpha_2 + ... + \alpha_k \neq 0, \alpha_i \in W_i$ and $\alpha_i = 0$

c) $\alpha_1 + \alpha_2 + ... + \alpha_k = 0, \alpha_i \in W_i$ and $\alpha_i \neq 0$

d) $\alpha_1 + \alpha_2 + ... + \alpha_k \neq 0, \alpha_i \in W_i$ and $\alpha_i = \alpha_j$

P.T.O.

v) Let $A = [T]_B$ then f is minimal poly. of T then f(T)=0 iff ________

a) A = 0 b) $A \neq 0$

c) $f(A) \neq 0$ d) f(A) = 0

b) Fill in the blanks : **5**

i) The unique monic generator of ideal $S(\alpha, w)$ is called __________

ii) If $\dim \cdot (v) = n$ and then subspace of $\dim \cdot n - 1$ is called ___________

iii) If V(F) is vector space and dim v=n S is subset of V s.t. S={0} then S°=_______

iv) N is linear operator on V(F) and dim v=n, N is nilpotent if ________

v) Let V be vector space. Let S be any subset of V,S is orthonormal set if ___

c) Whether the statements **true** or **false** : **4**

i) An orthogonal set with nonzero vector is linearly dependent.

ii) If V is vector space, T is linear operator on V. Then ristriction of T on W is also linear operator on V.

iii) The necessary condition for simulteneous digonalization is F be commutating family of operator.

iv) Let A be any $m \times n$ matrix over the field F. Then the row rank of A equal to column rank of A.

2. a) If V be a finite dimensional vector space of V then prove that A[A(W)]=W where W is any subspace of V.

b) Let V be a space of all polynomial function from R into R of the form $f(x) = c_0 + c_1 x + c_2 x^2 + c_3 x^3$ .The differentiation operator D maps V into V Let B={f_1,f_2,f_3} defined by fi(x)=x^{i-1} , i=1,2,3,4. Then find $[D]_B$.

c) Let V be a finite dimensional inner product space and T a linear operator on V. If T is invertible show that$T^*$is invertible and $(T^*)^{-1}=(T^{-1})^*$.

d) Define the term :

i) Orthonormal set

ii) Best approximation

iii) Orthogonal complement. **(4+4+3+3)**

3. a) If V be a finite dimensional vector space over F then show that each basis of V* is dual of some basis for V.

b) If V and W be finite dimensional vector space over the field F. Let B be orderd basis for V with dual basis B^* and Let B' be an orderd basis for W with dual basis B'^*. Let T be linear transformation from V into W. Let A be the matrix of T relative to B, B' and B be the matrix (of T^t) relative to B^*, B'^* then prove that, $B_{ij}=A_{ji}$. **(7+7)**

4. a) Prove that the minimal polynomial of a matrix or linear operator is divisor of every polynomial that anninilates the matrix or linear operator.

b) Let V be a finite dimensional vector space. Let $W_1,W_2,...,W_k$ then prove that following are equivalent

i) $W_1,W_2,..., W_k$ are independent.

ii) For each j , $2 \le j \le k$ we have $W_j \cap \sum_{i+j} W_i = \{0\}$.

iii) If B_i is orderd basis for $W_i, 1 \le i \le k$ then the sequence $B=\{B_1,B_2---,B_k\}$ is an orderd basis for W. **(7+7)**

5. a) Let V be an inner product space and, let $\beta_1, \beta_2, ..., \beta_n$ be an independent vectors in V. Then one may construct orthogonal vectors $\alpha_1, \alpha_2, ..., \alpha_n$ in V such that for each k=1,2,...,n the set $\{\alpha_1, \alpha_2, ..., \alpha_k\}$ is a basis for subspace spanned by $\beta_1, \beta_2, ... \beta_k$.

b) Let W be a finite dimensional subspace of an inner product space V and let E be the orthogonal projection of V on W. Then prove that E is an idempotent linear transformation of V on to W, W^1 is the null space of E and V= $W \oplus W^{\perp}$. **(7+7)**

6. a) If U be an linear operator on an innerproduct space V then prove that U is unitary if and only if the adjoint U^* of U exists and $U \cdot U^* = U^* U = I$.

b) If V be an finite dimensional inner product space and F a form on V. Then there is unique linear operator 'T' on 'V' such that f(α, β)=<T(α), β,> $\forall \alpha, \beta \in v$ and map $f \to T$ is an isomorphism of the space of form onto L(V,W). **(5+9)**

7. a) Explain Jordan Conanical Form, Nilpotent matrix and then find all possible Jordan Conanical Form of the matrix A whose characteristic polynomial is $f(x)=(x-7)^3(x-1)^2$.

b) In R^3 let α_1,(1,0,1), α_2=(0,1,–2), α_3=(-1,–1,0) If f is linear functional on R^3 such that f(α_1)=1, f(α_2)=1, f(α_3)=3 and if α=(a,b,c) find f(α). **(7+7)**

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Seat No.	

M.Sc. (Part – II) (Semester – III) Examination, 2014
MATHEMATICS (Paper – XIV)
Elective – II : Modeling and Simulation

Day and Date : Monday, 28-4-2014 Total Marks : 70
Time : 3.00 p.m. to 6.00 p.m.

***Instructions**: 1) Question no. **1** and **2** are **compulsory**.*
*2) Attempt **any three** questions from Q. **3** to Q. **7**.*
*3) Figures to **right** indicate **full** marks.*

1. A) Select the correct alternative. **10**

i) Simulation is

a) Descriptive in nature

b) Useful to analyze problem where analytical solution is difficult

c) Statistical experiments as such as its results are subject to statistical errors

d) All of the above

ii) Repetition of n independent Bernoulli trails reduced to

a) Poisson distribution b) Binomial distribution

c) Geometric distribution d) Hypergeometric distribution

iii) The slack for an activity in network is equal to

a) LS-ES b) LF-LS c) EF-ES d) EF-LS

iv) The activity which can be delayed without affecting the execution of immediate succeeding activity is determined by

a) Total float b) Free float

c) Independent float d) None of these

v) In M/M/1 : ∞/FCFS queue model if λ is mean customer arrival rate and μ is the mean service rate then the probability of server being busy is equal to

a) $\frac{\lambda}{\mu}$ b) $\frac{\lambda}{\mu - \lambda}$

c) $\frac{\mu}{\mu - \lambda}$ d) $\frac{\mu}{\lambda}$

vi) In queue model completely specified in the symbolic form (a/b/c/):(d/e), the last symbol 'e' specifies

a) The queue discipline b) The number of servers

c) The distribution of arrival d) The distribution of departure

vii) Time gap between placing of an order and its actual arrival in the inventory is known as

a) Lead time b) Demand

c) Both a) and b) d) None of the above

viii) Simulation of system in which the state changes smoothly with time are called

a) Discrete system b) Continuous system

c) Both a) and b) d) None of these

ix) PERT is used for the project involving activity of non-repetitive nature in which time estimation are

a) Certain b) Uncertain

c) Deterministic d) Both b) and c)

x) Markov chain said to be erotic chain if _________ of whose states are ergodic.

a) One b) Some

c) All d) None

B) Fill in the blanks. **4**

i) The long form of PERT is ________

ii) In inventory model, the number of unit required per period is called ________

iii) Chapman-Kolmogorov equation is Pij(t + T) = _________

iv) Let λ is arrival rate and μ is service rate, if $\lambda > \mu$ the queue is formed and ________ with time.

2. A) i) A manufacturing company purchases 9000 parts of a machine for its annual requirements, ordering one month usage at a time. Each part costs Rs. 20, the ordering cost per order is Rs. 15 and the carrying charges are 15% of the average inventory per year. You have been assigned to suggest a more economical purchasing policy for company. What advice would you offer and how much would it save the company per year ? **3**

ii) Write a note on queue configuration. **3**

B) i) What are the advantages and disadvantages of simulation ? **4**

ii) Explain the concept of anticipation inventory with example. **4**

3. A) Describe the deterministic inventory model of EOQ with uniform demand and no shortages. **7**

B) For various activity in the particular project the expected time (in days) of completions are as follow **7**

Activity	0–1	1–3	1–2	2–3	1–4	3–4	4–5
Duration	3	16	6	8	10	5	3

Draw a network diagram and identify the critical path.

4. A) Generate the five successive random number X_i, i = 1, 2, 3, 4, 5 by using $X_{i+1}=X_i * a$ (modulo m), starting with seed X_0 = 3 and parameters a = 7 and m = 15 (where m means that the number {Xi $*$ a} is divided by m repeatedly till the reminder is less than m). **7**

B) Define project duration, earliest event time, earliest start time, latest start time, earliest finish time in critical path computation. **7**

5. A) Define simulation. Write the advantages and limitations of simulation. **7**

B) The demand rate for a particular item is 12000 units/year. The ordering cost of Rs. 1,000 per order and the holding cost is Rs. 0.80 per month. If no shortage are allowed and the replacement is instantaneous the determine

i) Economic order quantity

ii) Number of order per year **7**

6. A) Explain the concept of inventory control. Write any four reasons for carrying inventories. **7**

B) Explain pure birth process. **7**

7. A) Differentiate between PERT and CPM. **7**

B) Explain the generation of random sample from continuous uniform distribution. **7**

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Seat No.	

M.Sc. II (Semester – III) Examination, 2014
MATHEMATICS (Paper – XV)
Numerical Analysis (Elective – III)

Day and Date : Wednesday, 30-4-2014 Max. Marks : 70
Time : 3.00 p.m. to 6.00 p.m.

Instructions : *1) Question No.* ***1*** *and* ***2*** *are* ***compulsory****.*
2) Attempt ***any three*** *question from question No.* ***3*** *to question No.* ***7****.*
3) Figures to the ***right*** *indicate* ***full*** *marks.*
4) ***Use*** *of calculator is* ***allowed****.*

1. A) Fill in the blanks (**one** mark **each**).

i) First divided difference of f(x) relative to x_0 and x_1 is ____________

ii) In false Position method, we choose two points x_0 and x_1 such that $f(x_0)$ and $f(x_1)$ are of ____________

iii) The Newton-Raphson method fails when $f'(x)$ is ____________

iv) The central difference operator $\delta^{3/2}$ is defined by the relation ____________

v) The effect of the error _________ with the order of the differences.

vi) The algebraic sum of the error in any difference column is ____________

vii) The value of y at x = 0.02 in solving $y' = -y$ by Euler method with the condition y(0) = 1 and h = 0.01 is ____________

B) Choose the correct alternative (**one** mark **each**). **6**

1) Which of the following is correct ?

a) $\nabla - \Delta = \Delta\nabla$ b) $\nabla + \Delta = -\Delta\nabla$ c) $\nabla - \Delta = -\Delta\nabla$ d) $\nabla + \Delta = \Delta\nabla$

2) If f(0) = 1, f(1) = 3, f(3) = 55 then the Lagrangian fundamental polynomial log(x) is

a) $\frac{1}{3}(x^2 - 4x + 3)$ b) $x^2 + 4x - 3$

c) $\frac{1}{2}(3x - x^2)$ d) $\frac{1}{6}(x^2 - x)$

P.T.O.

3) The convergence in bisection method is
   a) very slow b) quadratic c) cubic d) none
4) The relation between ∇ and E is
   a) $E = (1 - \nabla)^{-1}$ b) $E = (1 + \nabla)^{-1}$
   c) $\nabla = 1 + E^{-1}$ d) $\nabla = E - 1$
5) Simpson's $3/8$ rule for integration gives exact result when f(x) is a polynomial of degree
   a) 3 b) at most 3
   c) at least 3 d) none
6) If f(1) = 1, f(2) = 3 and f(3) = 5. What is the value of f(1.5) ?
   a) 1 b) 2 c) 2.5 d) 3
7) An approximate value of π is 3.142857 and it is true value is 3.1415926 then the absolute error E_n is
   a) 0.0012645 b) – 0.0012645
   c) 0.000402 d) – 0.000402

2. a) Prove that $\mu^2 = 1 + (1/4)\delta^2$. **3**

   b) Evaluate the sum $\sqrt{3} + \sqrt{5} + \sqrt{7}$ to significant digits and find its absolute and relative errors. **4**

   c) If $y_1 = 4$, $y_3 = 12$, $y_4 = 19$ and y_x 7 find x using Lagranges interpolation formula. **4**

   d) Write errors in Simpson's $1/3$ and Trapezoidal rule. **3**

3. a) Using iterative method find a-real root of the equation cosx = 3x – 1 correct to four decimal places. **7**

   b) Using Euler's method, solve the differential equation $\frac{dy}{dx} + 2y = 0$, y(0) = 1 take h = 0.1 and obtain y(0.1). **7**

4. a) Find root of the equation $x^3 - 2x - 5 = 0$ using Newton-Raphson method. **7**

   b) Solve the system of equations **7**

   $2x + 3y + z = 9$

   $x + 2y + 3z = 6$

   $3x + y + 2z = 8$

   By using factorization method.

5. a) Explain the errors in polynomial interpolation. **6**

   b) Reduce the matrix $\begin{bmatrix} 1 & 3 & 4 \\ 3 & 2 & -1 \\ 4 & -1 & 1 \end{bmatrix}$ to the tridiagonal form using Householder's method. **8**

6. a) Derive Newton's forward. Difference interpolation formula. **7**

   b) Evaluate the integral $\int_0^1 \frac{1}{1+x^2}\,dx$ by using Simpson's $1/3$ rule. Where the interval of integration is subdivided into 6 equal parts ? **7**

7. a) If $f(x) = 1/x$ then prove that $[x_0\, x_1 \ldots x_r] = \frac{(-1)^r}{x_0\, x_1 \ldots x_r}$. **6**

   b) Determine the value of y using modified Euler's method when x = 0.1 given that y(0) = 1 and $y' = x^2 + y$, take h = 0.05, with $x_0 = 0$, $y_0 = 1.0$. **8**

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SLR-VA – 16

Seat No.	

M.Sc. (Part – II) (Semester – IV) Examination, 2014
MATHEMATICS (Paper – XVI)
Measure and Integration

Day and Date : Tuesday, 22-4-2014 Total Marks : 70

Time : 3.00 p.m. to 6.00 p.m.

Instructions : *1) Q. **1** and Q. **2** are **compulsory**.*
*2) Attempt **any three** questions from Q. **3** to Q. **7**.*
*3) Figures to the **right** indicate **full** marks.*

1. **Type I :** Fill in the blanks (**one** marks **each**) : **5**

1) Let μ be a measure on an algebra $\mathcal{A}$. The outer measure induced by μ is defined by $\mu^*(E)$ = ______________

2) A collection of subsets of X forms a σ-algebra if ____________

3) A measure μ is called semifinite if ____________

4) A measure set E is called a null set if ___________

5) μ and ν are measures on $(X, \mathcal{B})$. Then $\nu << \mu$ if _____________

Type II : State **true** or **false** (**one** mark **each**) : **5**

1) Lebesgue measure is not complete.

2) In Fubini's theorem function f defined $X \times Y$ is measurable and integrable.

3) Every semialgebra is an algebra.

4) Jordan decomposition is unique.

5) Set of all μ^*-measurable sets forms a σ-algebra.

P.T.O.

Type III : Tick mark (✓ the correct alternative (**one** mark **each**) : **4**

1) A set of A is called negative w.r.t. a signed measure ν if

   a) $\nu(A) \leq 0$ and finite

   b) $\nu(A) \leq 0$ and need not be finite

   c) $\nu(A) \leq 0$ and there is a subset with negative measure

   d) $\nu(A) \leq 0$ and every measurable subset has non positive measure

2) If f is an integrable function defined on a measurable set E such that f = 0 a.e. then

   a) $\int_E f \, d\mu \neq 0$
   b) $\int_E f \, d\mu = 0$

   c) $\int_E f \, d\mu \geq 0$
   d) $\int_E f \, d\mu$ does not exists

3) Let μ and ν be two measures on a measurable space. Consider the two statements

   (I) $\mu \perp \nu \Rightarrow \nu \perp \mu$

   (II) $\mu << \nu \Rightarrow \nu << \mu$

   Then

   a) Only (I) is true
   b) Only (II) is true
   c) Both (I) and (II) are true
   d) Both (I) and (III) are false

4) Consider the two statements

   (I) [0, 1] is an uncountable set in IR

   (II) Lebesgue measure of [0, 1] is infinite

   a) Only (I) is true
   b) Only (II) is true
   c) Both (I) and (II) are true
   d) Both (I) and (II) are false

2. a) Let $(X, \mathcal{B}, \mu)$ be a measure space : If $A \subseteq B$ then show that $\mu(A) \leq \mu(B)$, $A, B \in \mathcal{B}$.

   b) If μ is a complete measure and $\mu(E, \Delta E_2) = 0$ and $E_1 \in \mathcal{B}$ then show that $E_2 \in \mathcal{B}$.

   c) Define a product measure on a product space $X \times Y$.

   d) Show that for any set E, $\mu_*(E) = \mu^*(E)$. **(3+4+3+4)**

3. a) Let $(X, \mathscr{B}, \mu)$ be a measure space. Let $\{E_i\}_{i=1}^{\infty}$ be a sequence of sets in $\mathscr{B}$ such that $\mu(E_1) < \infty$ and $E_i \supset E_{i+1}$ for all $i \in N$. Prove that

$$\mu\left(\bigcap_{i=1}^{\infty} E_i\right) = \lim_{i \to \infty} \mu(E_i).$$

b) If μ is a complete measure and f is a measurable function then prove that $f = g$ a. e $[\mu]$ implies g is measurable. **(8+6)**

4. a) If f and g are nonnegative measurable functions defined on a measure space $(X, \mathscr{B}, \mu)$, prove that

i) $f \le g$ a.e $\Rightarrow \int f \, d\mu \le \int g \, d\mu$

ii) $\int c f \, d\mu = c \int f \, d\mu$, where $c > 0$.

b) State and prove Lebesgue convergence theorem. **(6+8)**

5. a) Show that Hahn decomposition is unique except for null sets.

b) State and prove Jordan decomposition theorem. **(6+8)**

6. a) Show that the condition of σ-finiteness is essential in Radon-Nikodym theorem.

b) Let E be a measurable subset of a product space $X \times Y$ such that $(\mu \times \nu)(E) = 0$. Prove that $\nu(E_x) = 0$ a.e , $x \in X$. **(7+7)**

7. a) Let μ be a measure on an algebra $\mathscr{A}$ and μ^* be the outer measure induced by μ. If $A \in \mathscr{A}$ then prove that A is measurable w.r.t μ^*.

b) If E and F are disjoint sets, then prove that

$\mu_*(E) + \mu_*(F) \le \mu^*(E \cup F) \le \mu_*(E) + \mu^*(F)$. **(6+8)**

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Seat No.	

M.Sc. (Part – II) (Semester – IV) Examination, 2014
MATHEMATICS (Paper – XVII)
Partial Differential Equations

Day and Date : Thursday, 24-4-2014 Max. Marks : 70
Time : 3.00 p.m. to 6.00 p.m.

Instructions *: 1) Question No.* ***1*** *and* ***2*** *are* ***compulsory****.*
2) Attempt ***any three*** *questions from Q. No.* ***3*** *to Q. No.* ***7****.*
3) Figures to the ***right*** *indicate* ***full*** *marks.*

1. A) Fill in the blanks (**one** mark **each**) :

1) A two parameter family of solutions z = F(x, y, a, b) is called complete integral of the p.d.c. f(x, y, z, p, q) = 0, if in the region considered, the rank of the matrix __________ is two.

2) The general solution of the p.d.c. xp + yq = z is ____________

3) Write true or false : There does not exists an integrating factor for a Plaffian differential equation in two variables.

4) The one dimensional wave equation is given by ___________

5) A second order p.d.c. $Ru_{xx} + Su_{xy} + Tu_{uy} + 9(x, y, u, u_x, u_y) = 0$ is called parabolic if __________

6) If $f(x, y, z) = x^2 + y^2 + z^2$ then $\nabla^2 f$ = __________

7) Write true or false : The solution of Dirichlet problem if it exists need not be unique.

B) Choose the correct answer (**one** mark **each**) :

1) Complete integral of the equation z = px + qy + log pq is

a) z = ax + by + ab   b) z = ax + by + log ab

c) z = a + b ab xy   d) $z = ax + by + \dfrac{a^2 + b^2}{ab}$

2) Eliminating arbitrary function F from $z = F\left(\sqrt{x^2 + y^2}\right)$ gives rise to p.d.c.

a) $yp - xq = 0$ b) $p - q = 0$

c) $p^2 + q^2 = 0$ d) $x^2p - y^2q = 0$

3) If there is a functional relation between two functions u(x, y) and v(x, y) not involving x and y explicitly then

a) $\frac{\partial u}{\partial x} = 0$ and $\frac{\partial v}{\partial y} \neq 0$

b) $\frac{\partial v}{\partial x} = 0$ and $\frac{\partial u}{\partial y} \neq 0$

c) $\frac{\partial(u, v)}{\partial(x, y)} = 0$

d) $\frac{\partial(u, v)}{\partial(x, y)} \neq 0$

4) A function $\phi(x, y)$ is called harmonic if it satisfies

a) Wave equation

b) Laplace equation

c) Heat equation

d) Lagranges equation

5) A p.d.c. $e^{2x} u_{xx} + 2e^{x+y} u_{xy} + e^{2y} u_{yy} = 0$ is of

a) Parabolic type

b) Hyperbolic type

c) Elliptic type

d) Elliptic and hyperbolic type

6) The condition that the surfaces f(x, y, z) = c forms a family of equipotential surfaces is that

a) $\frac{\nabla^2 f}{|\nabla f|^2} = 0$ b) $\frac{\nabla f}{|\nabla^2 f|^2} = 0$

c) $\frac{\nabla^2 f}{|\nabla f|^2}$ is function of f only   d) $\frac{\nabla^2 f}{|\nabla f|^2}$ is not the function of f

7) The two solutions of a Neumann problem is differ by

a) function of x   b) function of y

c) function of x and y   d) constant

2. a) Let u(x, y) and v(x, y) be two functions of x and y such that $\frac{\partial v}{\partial y} \neq 0$. If further $\frac{\partial(u, v)}{\partial(x, y)} = 0$, then prove that there exists a relation between u and v not involving x and y explicitly.

b) Find the general integral of

$x^2p + y^2q = (x + y)\, z$.

c) Show the necessary condition for the existence of the solution of the Neumann problem $\nabla^2 u = 0$ in D and $\frac{\partial u}{\partial n} = f(s)$ on B, is that the integral of f over the boundary B should vanish. Here B is the boundary of the region D.

d) Show that the surfaces $f(x, y, z) = x^2 + y^2 + z^2 = c$, $c > 0$, can form an equipotential family of surfaces. **(4, 3, 4, 3)**

3. a) If the partial differential equations $f(x, y, z, u_x, u_y, u_z) = 0$ and $h(x, y, z, u_x, u_y, u_z) = 0$ are compatible then prove that

$$\frac{\partial(f, h)}{\partial(x, u_x)} + \frac{\partial(f, h)}{\partial(y, u_y)} + \frac{\partial(f, h)}{\partial(z, u_z)} = 0$$

b) Obtain d'Alembert's solution of the one dimensional wave equation which describes the vibrations of an infinite string. **(7, 7)**

4. a) Solve $y_{tt} - c^2 y_{xx} = 0,\ 0 < x < 1,\ t > 0$
Subject to $y(0, t) = y(1, t) = 0,\ t > 0,$
and $y(x, 0) = 0,\ y_t(x, 0) = x^2,\ 0 \le x \le 1.$

b) Find the complete integral of the p.d.e. $f = (p^2 + q^2)\, y - qz = 0$ by charpits method. **(7, 7)**

5. a) By Jacobi's method, solve the equation $z^2 + zu_z - u_x^2 - u_y^2 = 0$.

b) Find the solution of the problem

$\nabla^2 u = 0,\ -\infty < x < \infty,\ y > 0$

$u(x, 0) = f(x),\ -\infty < x < \infty$

Such that u is bounded as $y \to \infty$, u and u_x vanish as $|x| \to \infty$. **(7, 7)**

6. a) Find the integral surface of the differential equation $(p^2 + q^2)\, x = pz$, passing through the curve C : $x_0 = 0,\ y_0 = s^2,\ z_0 = 2s$.

b) Suppose that u(x, y) is harmonic in a bounded domain D and continuous in $\overline{D} = D \cup B$, then prove that u attains its maximum on the boundary B of D. **(7, 7)**

7. a) Show that the solution for the Dirichlet problem for a circle is given by the Poisson integral formula.

b) Show that the equations $xp - yq - x = 0,\ x^2 p + q - xz = 0$ are compatible and find a one-parameter family of common solutions. **(7, 7)**

Seat No.	

M.Sc. II (Semester – IV) Examination, 2014
MATHEMATICS (Elective – I)
Integral Equations (Paper – XVIII)

Day and Date : Saturday, 26-4-2014 Total Marks : 70
Time : 3.00 p.m. to 6.00 p.m.

Instructions : *1) Q. No. **1** and **2** are **compulsory**.*
*2) Attempt **any three** questions from Q. No. **3** to Q. No. **7**.*
*3) Figures to the **right** indicate **full** marks.*

1. A) Choose correct alternative : **7**

1) The equation $F(x) = \lambda \int_a^b k(x, t)\, y(t)\, dt = 0$ is known as

a) Fredholm integral equation of second kind

b) Fredholm integral equation of first kind

c) Homogeneous Fredholm integral equation of second kind

d) None of these

2) Conversion of differential equation $y'' + y = 0$ when $y(0) = y'(0) = 0$ into integral equation is

a) $y(x) = \int_0^x (x - t)\, y(t)\, dt$ b) $y(x) = \int_0^x x \cdot y(t)\, dt$

c) $y(x) = -\int_0^x x \cdot y(t)\, dt$ d) $y(x) = -\int_0^x (x - t)\, y(t)\, dt$

3) When an ordinary differential equation is to be solved under condition involving dependent variable and it's derivative at two different value of independent variable then the problem under consideration is said to be ?

a) Boundary value problem b) Initial value problem

c) Both a and b d) None of these

4) The inhomogeneous Fredholm integral equation with a separable kernel has
   a) Many solution
   b) Unique solution
   c) No solution
   d) Trivial solution

5) The kernel k(s, t) is said to be degenerate if $k(s, t) = \sum_{i=1}^{n} ai(s) \,.\, bi(t)$ where ai(s) and bi(t) are
   a) zero
   b) non zero
   c) linearly dependent
   d) linearly independent

6) If the boundary value problem is self adjoint then green function G(x, t) is
   a) Symmetric
   b) Non-symmetric
   c) Separable
   d) Zero

7) The IVP $y'' + y = 0, y(0) = 0, y'(0) = 1$ can be converted into
   a) Volterra integral equation
   b) Fredholm integral equation of first kind
   c) Fredholm integral equation of second kind
   d) None of these

B) State whether the statement are **true** or **false** : **7**

1) A homogeneous Fredholm integral equation of second kind may generally have no eigen values.
2) Any integral equation can be solved by using Laplace equation.
3) Every initial value problem can be converted into Volterra integral equation of first kind.
4) All eigen values of symmetric kernel are real.
5) $Y(x) = \lambda \int_a^b k(x, t)\, y(t)\, dt$ then if y(x) eigen function then c. y(x) is also eigen function (Where c is arbitrary constant).

6) $\int_a^b |y(x)|^2 dx < \infty$ then y(x) is L_2 function.

7) Volterra integral equation of first kind can be converted into Volterra integral equation of second kind.

2. a) Show that the function $y(x) = (1 + x^2)^{-3/2}$ is a solution of the

$$y(x) = \frac{1}{1+x^2} - \int_0^x \frac{t}{1+x^2} y(t)\, dt.$$ **4**

b) Convert the following differential equation into an integral equation

$$y'' + \lambda xy = F(x)$$

$y(0) = 1,\ y'(0) = 0.$ **4**

c) Define Green function. **3**

d) Give types of kernel. **3**

3. a) Obtain the Fredholm integral equation corresponding to BVP, $\frac{d^2y}{dx^2} + \lambda y = x$ with boundary condition $y(0) = 0,\ y'(1) = 0.$ **7**

b) By using reduction method solve $y(x) = x + \lambda \int_0^1 (xt^2 + tx^2)\, y(t)\, dt.$ **7**

4. a) Find the Resolvent kernel of integral equation with the kernel $k(x, t) = e^{x-t}$. **7**

b) State and prove Hilbert Schmidth theorem. **7**

5. a) Using method of approximation solve, $y(x) = 1 + \lambda \int_0^1 xt\, y(t)\, dt.$ **7**

b) Determine the Green's function of the BVP,

$y''(x) + \lambda\, y(x) = 0, \quad y(0) = 0$

$y'(1) + \gamma_2 \cdot y(1) = 0.$ **7**

6. a) Show that the eigen function of a symmetric kernel corresponding to different eigen values are orthogonal. **7**

b) Find the eigen values and the corresponding eigen function of the integral equation $y(x) = \lambda \int_0^1 (2xt - 4x^2)\, y(t)\, dt$. **7**

7. a) Solve the integral equation by using Fourier transform

$$\int_0^\infty F(x) \cos px\, dx = \begin{cases} 1 = p \ ; & 0 \le p \le 1 \\ 0 \ ; & p > 1 \end{cases}.$$ **7**

b) Solve homogeneous Fredholm integral equation

$$y(x) = \lambda \int_0^1 e^x \; e^t \, y(t)\, dt.$$ **7**

Seat No.	

M.Sc. (Part – II) (Semester – IV) Examination, 2014
MATHEMATICS (Paper – XIX)
Elective – II : Operations Research

Day and Date : Tuesday, 29-4-2014 Total Marks : 70
Time : 3.00 p.m. to 6.00 p.m.

Instructions : *1) Attempt **five** questions.*
*2) Q. No. (**1**) and Q. No. (**2**) are **compulsory**.*
*3) Attempt **any three** from Q. No. (**3**) to Q. No. (**7**).*
*4) Figures to the **right** indicate **full** marks.*

1. A) Select correct alternative : **5**

1) A necessary and sufficient condition for a basic feasible solution to a minimization LPP to an be optimum is that (for all j)

a) $z_j - c_j \geq 0$ b) $z_j - c_j = 0$
c) $z_j - c_j \leq 0$ d) $z_j - c_j < 0$ or $z_j - c_j < 0$

2) Dual simplex method is applicable to those LPPs that start with

a) an infeasible solution
b) an infeasible but optimum solution
c) a feasible solution
d) a feasible and optimum solution

3) Consider the LPP : Maximize Z = 3x + 5y; subject to $x + 2y \leq 4$, $2x + y \geq 6$ and $x \geq 0$ and $y \geq 0$. This problem represents a

a) Zero – one IPP b) Pure IPP c) Mixed IPP d) Non – IPP

4) For a two person game with A and B, the minimizing and maximizing players, the optimum strategies are

a) Minimax for A and maximin for B
b) Maximax for A and minimax for B
c) Minimin for A and maximin for B
d) Maximin for A and minimax for B

P.T.O.

5) In two phase simplex method phase I
   a) gives a starting basic feasible solution
   b) optimize the objective function
   c) provides optimal solution
   d) none of these

B) Fill in the blanks : **5**

1) In cutting plane algorithm, each cut involves the introduction of ________
2) The solution of m basic variables when each of the n-m non-basic variables is set to zero is called as ________
3) For an unbounded primal problem, the dual would be ____________
4) In a simplex method, all entries in the key column $y_{ir} \leq 0$ then there exist ____________ solution to given problem.
5) If $X^T QX$ is said to be negative semi definite if ______________

C) State whether the following statements are **true** of **false** : **4**

1) Linear programming is probabilistic in nature.
2) An LPP with all its constraints are of the type "$\leq$" is said to be in canonical form.
3) The solution to maximization LPP is not unique if $(z_j - c_j) > 0$ for each of the non-basic variables.
4) Dual simplex method is applicable to an LPP if initial basic feasible solution is not optimum.

2. a) Answer the following : **6**

1) Give the general rules for converting any primal into its dual.
2) Define : solution, feasible solution and basic solution of an LPP.

b) Answer the following : **8**

1) Let S, T be two convex sets in R^n then prove that $\alpha S + \beta T$ is also convex $(\alpha, \beta \in R)$.
2) State the formulae to obtain the outgoing and incoming vector in dual simplex method.

3\. a) Explain simplex algorithm to solve the linear programming problem. **(8+6)**

b) Explain the penalty method for solving a given LPP.

4\. a) Prove that dual of the dual is primal

b) Use dual simplex method to solve the following LPP :

Minimize $Z = -2x_1 - x_2$

Subject to the constraints,

$2x_1 + x_2 \geq 3,\ 4x_1 + 3x_2 \geq 6,\ x_1 + 2x_2 \geq 3$

and $x_1, x_2 \geq 0$ **(6+8)**

5\. a) Show that solving of two person zero-sum game is equivalent to solving a LPP.

b) Explain the graphical method for solving 2×m game. Solve the game with following pay-off matrix using graphical technique.

$$A\begin{bmatrix} 2 & 4 & 3 \\ 1 & 2 & 6 \end{bmatrix}.$$ **(6+8)**

6\. a) Describe Gomory's method of solving an all integer LPP.

b) Solve the LPP by Big-M method

Maximize $Z = -2x_1 - x_2$

Subject to the constraints,

$3x_1 + x_2 = 3,\ 4x_1 + 3x_2 \geq 6,\ x_1 + 2x_2 \leq 4$

and $x_1, x_2 \geq 0$. **(6+8)**

7\. a) Derive Kuhn-Tucker conditions for an optimum solution to a Quadratic programming problem.

b) Use Beal's method to solve

Maximize $Z_x = 4x_1 + 6x_2 - 2x_1^2 - 2x_1x_2 - 2x_2^2$

Subject to the constraints,

$x_1 + 2x_2 \leq 2$ and $x_1, x_2 \geq 0$. **(6+8)**

SLR-VA – 2

Seat No.	

M.Sc. (Part – I) (Semester – I) Examination, 2014
MATHEMATICS
Algebra – I (P. No. II)

Day and Date : Wednesday, 23-4-2014 Total Marks : 70
Time: 11.00 a.m. to 2.00 p.m.

__Instructions__ : 1) Q. __1__ and Q. __2__ are __compulsory.__
2) Attempt __any three__ questions from Q. __3__ to Q. __7.__
3) Figures to the __right__ indicates __full__ marks.

1. **Type I** : Define the following (**two** marks **each**) :

i) Solvable group

ii) Maximal ideal

iii) Irreducible polynomial

Type II: State whether the following statements are **True** or **False** (**one** mark **each**) :

iv) Every prime ideal is maximal.

v) $1 + x + x^2 + ... + x^{P-1}$ is irreducible over $\mathbb{Z}$.

vi) A group of order | 2 | is abelian.

vii) Product of two primitive polynomials is primitive.

Type III : Fill in the blanks (**one** mark **each**) :

viii) R/I is a field iff I is ________________

ix) A sylon 3-subgroup of a group of order 12 has order ________________

x) Units in the ring $\mathbb{Z}$[i] are precisely ________________

xi) Subgroup of a solvable group is ________________

2. a) Let G be a nilpotent group. Show that every subgroup of G is nilpotent.

   b) Show that an abelian group G has a composition series iff G is finite.

   c) Show that the polynomial $x^3 + 3x + 2$ is irreducible over $\mathbb{Z}_5$.

   d) Show that the Kernel of a homomorphism is a submodule. **(3+4+3+4)**

3. a) Let H be a subgroup of G and N be a normal subgroup of G then show that $HN/N \simeq H/H \cap N$.

   b) Let X be a G-set and $x \in X$. Show that $|xG| = (G : G_x)$. **(7+7)**

4. a) Show that a group of order 48 is not simple.

   b) Prove that in an integral domain D, there exists l.c.m. of any two non-zero elements and it is unique apart from the distinction between associates. **(7+7)**

5. a) Prove that any two subnormal series of a group have isomorphic refinements.

   b) State and prove Third Sylow Theorem. **(7+7)**

6. a) Let D be an integral domain with unity. Show that two non zero elements $a, b \in D$ are associates iff $a|b$ and $b|a$.

   b) Show that every PID is a UFD. **(7+7)**

7. a) If G is a group and N is a normal subgroup of G such that both N and G/N are solvable. Prove that G is solvable.

   b) Prove that every finite integral domain is a field.

   c) If F is a field, show that every ideal in F [X] is principal ideal. **(5+4+5)**

SLR-VA – 20

Seat No.	

M.Sc. (Part – II) (Semester – IV) Examination, 2014
MATHEMATICS (Paper – XX)
Elective – III : Probability Theory

Day and Date : Friday, 2-5-2014 Total Marks : 70
Time : 3.00 p.m. to 6.00 p.m.

Instructions : *1) Attempt **five** questions.*
*2) Q. No. **1** and Q. No. **2** are **compulsory**.*
*3) Attempt **any three** from Q. No. **3** to Q. No. **7**.*
*4) Figures to the **right** indicate **full** marks.*

1. A) Select correct alternative : **5**

1) Let $F_1 = \{A, A^c, \phi, \Omega\}$ and $F_2 = \{B, B^c, \phi, \Omega\}$ then
   a) Only F_1 is field b) Only F_2 is filed
   c) $F_1 \cap F_2$ is field d) $F_1 \cup F_2$ is field

2) Two random variables X and X' are equivalent if
   a) $P(X = X') = 1$ b) $X_n \xrightarrow{P} X$ and $X_n \xrightarrow{P} X'$
   c) $X_n \xrightarrow{a.s} X$ and $X_n \xrightarrow{a.s} X'$ d) all the above

3) $I(A_1 \cup A_2) =$ ______________
   a) $I(A_1)$ b) $I(A_2)$
   c) Min $\{A_1, A_2\}$ d) Max $\{A_1, A_2\}$

4) Let $\{A_n\}$ be a sequence of sets then
   a) $\lim A_n$ always exists b) $\underline{\lim}\, A_n$ and $\overline{\lim}\, A_n$ always exists
   c) $\overline{\lim}\, A_n \subseteq \underline{\lim}\, A_n$ d) none of these

5) Characteristic function of Binomial random variable with parameters n and p is
   a) $(p + qe^{it})^n$ b) $(pe^{it} + q)^n$
   c) $(p - qe^{it})^n$ d) $(pe^{it} - q)^n$

P.T.O.

B) Fill in the blanks : **5**

1) The range of indicator function I(A) is ____________

2) Let $\{A_n)$ be a sequence of events such that $\sum_{n=1}^{\infty} P(A_n) < \infty$ then $P\left(\overline{\lim} A_n\right) =$ __________

3) If the characteristic function is real then X is _________

4) A countable linear combination of indicator functions is ____________ function.

5) Characteristic function uniquely determines the ______________

C) State whether the following statements are **true** or **false** : **4**

1) A field of subsets of a set Ω is always a σ – filed.

2) Convergence in r^{th} mean is stronger than convergence in probability.

3) Union of fields is a filed.

4) Minimal field is unique.

2. a) Answer the following : **6**

1) Show that X is integrable iff |X| is integrable.

2) If $A_n \rightarrow A$ show that $A_n^c \rightarrow A^c$.

b) Write short notes on the following : **8**

1) Convergence in distribution

2) Strong Law of Large Numbers (SLLN).

3. a) Define lim inf and lim sup of a sequence of sets. With usual notations show that

$$\overline{\lim} \ (A_n \cup B_n) = \overline{\lim} \ A_n \cup \overline{\lim} \ B_n$$

b) Let A_n be set of points (x, y) of Cartesian plane lying within the rectangle bounded by two axes and the lines x = n and y = $\frac{1}{n}$ Find lim A_n if exists. **(8+6)**

4. a) Define probability measure. Prove that if P and Q are probability measures then $P^*(A) = \alpha\ P(A) + (1 - \alpha)\ Q(A)$, $0 \leq \alpha \leq 1$ is a probability measure.

b) If $A_n \uparrow A$ prove that $P(A_n) \uparrow P(A)$, where P is probability measure. **(8+6)**

5. a) State and prove monotone convergence theorem.

b) If $X_n \xrightarrow{r} X$ show that $E|X_n|^r \longrightarrow E|X|^r$ **(7+7)**

6. a) Define pair wise and mutual independence of events.

b) If X and Y are arbitrary independent random variables then show that $E(XY) = E(X)\,E(Y)$.

c) If ϕ is characteristic function of random variable X then find characteristic function of $2X - 1$. **(5+6+3)**

7. a) State :

i) Liapounov's CLT

ii) Lindeberg-Feller CLT.

Show that Liapounov's condition for CLT implies Lindeberg's condition.

b) State inversion formula and obtain the distribution of random variable corresponding to characteristic function $e^{-|t|}$. **(8+6)**

Seat No.	

M.Sc. (Part – I) (Semester – I) Examination, 2014
MATHEMATICS (Paper – III)
Real Analysis – I

Day and Date : Friday, 25-4-2014 Total Marks : 70

Time : 11.00 a.m. to 2.00 p.m.

Instructions : *i) Q. No.* ***1*** *and* ***2*** *are* ***compulsory****.*
ii) Attempt ***any three*** *from Q. No.* ***3*** *to* ***7****.*
iii) Figures to the ***right*** *indicate* ***full*** *marks.*

1. A) Fill in the blanks (**One** mark **each**) :

1) A bounded function f is integrable if the set of points of discontinuity has only a finite number of ____________

2) The Riemann stieltjes integral reduces to Riemann integral when ________

3) The total derivative of a ___________ function is the function itself.

4) __________ integral is the greatest lower bound of the set of upper sums.

B) State whether **true** or **false** (**One** mark **each**) :

1) The function $f(x) = \begin{cases} 1, & \text{when x is rational} \\ -1, & \text{otherwise} \end{cases}$ is bounded and integrable.

2) Every continuous function is integrable.

3) The oscillatory sum U(P, f) – L(P, t) is always nonnegative.

4) If $\bar{f} = IR^n \to IR^m$ then $D_{1,2}\bar{f}(x, y) = D_{2,1}\bar{f}(x, y)$.

C) Define the following (**two** marks **each**) :

1) Riemann sum

2) Partition of [a, b]

3) Norm of partition.

2. a) Prove with the help of example that the equation $\int_a^b f'(x)\,dx = f(b) - f(a)$, is not always valid.

b) If f is bounded and integrable on [a, c], [c, b] then prove that it is integrable on [a, b] where c is point of [a, b].

c) Assume $\bar{f}$ is differentiable at $\bar{c}$ with total derivative $T_{\bar{c}}$. Then prove that directional derivative $\bar{f}'(\bar{c}\,;\bar{u})$ exists for every $\bar{u}$ in IR^n and $T_{\bar{c}}(\bar{u}) = \bar{f}'(\bar{c}\,;\bar{u})$.

d) Let $\bar{f} : IR^2 \to IR^3$ be defined by the equation f(x, y) = (sinx cosy, sinxsiny, cosxcosy). Determine the Jacobian matrix $D\bar{f}(x, y)$. **(4+4+3+3)**

3. a) If P* is a refinement of P, then prove that $U(P^*, f, \alpha) \le U(P, f, \alpha)$.

b) State and prove Riemann-Lebesque lemma. **(5+9)**

4. a) If f_1, f_2 are two bounded and integrable functions on [a, b] then prove that $f = f_1 - f_2$ is also integrable on [a, b] and $\int_a^b f\,dx = \int_a^b f_1\,dx - \int_a^b f_2\,dx$.

b) Compute $\int_{-1}^{1} f\,dx$ where f(x) = |x|. **(9+5)**

5. a) If f and g are integrable on [a, b] and g keeps the same sign over [a, b], then prove that there exists a number μ lying between the bounds of f such that $\int_a^b fg\,dx = \mu \int_a^b g\,dx$.

b) If f is bounded and integrable on [a, b], then prove that | f | is also bounded and integrable on [a, b]. Moreover $\left| \int_a^b f dx \right| \le \int_a^b |f| \, dx$. **(6+8)**

6. a) Let S be an open subset of IR^n and assume that $\bar{f} : S \to IR^n$ is differentiable at each point of S. Let $\bar{x}$ and $\bar{y}$ be two points in S such that $L(\bar{x}, \bar{y}) \subseteq S$. Then prove that for every vector $\bar{a} \in IR^m$ there is a point $\bar{z}$ in $L(\bar{x}, \bar{y})$ such that $\bar{a} \cdot \{\bar{f}(\bar{y}) - \bar{f}(\bar{x})\} = \bar{a} \cdot \{\bar{f}'(\bar{z})\,(\bar{y} - \bar{x})\}$.

b) Assume $\bar{g}$ is differentiable at $\bar{a}$, with total derivative $\bar{g}'(\bar{a})$. Let $\bar{b} = \bar{g}(\bar{a})$ and assume that $\bar{f}$ is differentiable at $\bar{b}$, with total derivative $\bar{f}'(\bar{b})$. Then prove that composite function $\bar{h} = \bar{f} \circ \bar{g}$ is differentiable at $\bar{a}$ and find its total derivative. **(6+8)**

7. a) State and prove inverse function theorem.

b) If f = u + iv is a complex valued function with a derivative at a point z in C, then prove that $J_f(z) = |f'(z)|^2$. **(10+4)**

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Seat No.	

M.Sc. (Part – I) (Semester – I) Examination, 2014
MATHEMATICS
Differential Equation (Paper No. IV)

Day and Date : Monday, 28-4-2014 Max. Marks : 70
Time : 11.00 a.m. to 2.00 p.m.

Instructions : *1) Q. No.* ***1*** *and Q. No.* ***2*** *are* ***compulsory****.*
2) Attempt ***any three*** *questions from Q. No.* ***3*** *to Q. No.* ***7****.*
3) Figures to the ***right*** *indicate* ***full*** *marks.*

1. A) Choose the correct alternative (**one** mark **each**). **6**

1) For a Bessel's function $J_{-n}(x) =$

a) $(-1)^n J_n(x)$ b) $(-1)^{n+1} J_n(x)$
c) $(-1)^n J_{n+1}(x)$ d) $(-1)^{n+1} J_{n+1}(x)$

2) The generating function for Jn(x) is

a) $e^{\frac{x}{2}\left(z+\frac{1}{z}\right)}$ b) $e^{-\frac{x}{2}\left(z+\frac{1}{z}\right)}$ c) $e^{\frac{x}{2}\left(z-\frac{1}{z}\right)}$ d) $e^{-\frac{x}{2}\left(z-\frac{1}{z}\right)}$

3) The expression for Legendre's poly. $P_n(x)$

a) $\frac{1}{2^n n!} \frac{d^n}{dx^n} (x^2+1)^n$ b) $\frac{d}{2^n n!} \frac{d^n}{dx^n} (x^2-1)^n$

c) $\frac{1}{2^n n!} \frac{d^n}{dx^n} (x+1)^n$ d) $\frac{1}{2^n.n!} \frac{d^n}{dx^n} (x-1)^n$

4) Bessel's function of zero order and first kind is given by, $J_0(x) =$ ________

a) $\sum_{m=0}^{\infty} \frac{(-1)^m}{(m!)^2} \left(\frac{x}{2}\right)^{2m}$ b) $\sum_{m=0}^{\infty} \frac{(-1)^{m+1}}{(m!)^2} \left(\frac{x}{2}\right)^{2m}$

c) $\sum_{m=0}^{\infty} \frac{(-1)^{m+1}}{(m!)^2} \left(\frac{x}{2}\right)^{m}$ d) $\sum_{m=0}^{\infty} \frac{(-1)^m}{(m!)^2} \cdot \left(\frac{x}{2}\right)^{m}$

5) The general solution of $y'' - 4y = 0$ is

a) $C_1 e^{2x} + C_2 e^{2x}$ b) $C_1 e^{2x} + C_{2x} e^{2x}$

c) $C_1 e^{-2x} + C_2 e^{-2x}$ d) $C_1 e^{-2x} + C_2 . x . e^{-2x}$

6) An eq^n of the form, C_0 (x) $(x - x_0)^n . y^{(n)}$ + C_1 (x) $(x - x_0)^{n-1} . y^{(n-1)}$ + __________ + C_n (x) y = 0 has regular singular point at x_0 if C_0, C_1 ...... C_n are __________ at x = x_0 and $C_0(x_0)$ is

a) Analytic, zero b) not analytic, zero

c) Analytic, non-zero d) not analytic non-zero

B) Fill in the blanks (**one** mark **each**) : **8**

1) Legendre's differential equation is given by L (y) = _________

2) The characteristic polynomial of, $y''' - 3y' - 2y = 0$ is p(r) = _________

3) F (x, y) satisfies Lipschitz condition on S if there exist a constant K>O such that , _________

4) The indicial polynomial for $x^2 y'' + a(x). \, xy' + b(x).y = 0$ is q (r) = _________

5) Two functions $\phi_1, \ \phi_2$ are said to be linearly independent iff $W(\phi_1, \phi_2)$ (x) is _________

6) The general solution of $3y'' + 2y' = 0$ is ϕ (x) = _________

7) Alinear diff. equation L (y) = b (x) is said to homogeneous if b(x) = _________

8) Regular Singular point of Bessel's equation is x = _________

2. a) Show that Wronskian of functions x^2 and $x^2 \log x$ is non-zero (x>0). **4**

b) Find solution of, $y''' - i.y'' + 4y' - 4iy = 0$ **3**

c) Prove or disprove x = 0 is regular singular point of equation,

$3x^2 y'' + x^6 . y' + 2xy = 0$. **3**

d) Consider the equation $y'' + \alpha(x).y = 0$ where α is continuous function on $-\infty < x < \infty$. Let $\phi_1, \ \phi_2$ be the bases for the solutions satisfying,

$\phi_1(0) = 1, \ \ \phi_2(0) = 0, \ \ \phi_1^1(0) = 1, \ \phi_2^1(0) = 1$. Show that $W(\phi_1, \phi_2)$ (x) = 1 for all x. **4**

3. a) One solution of $x^2y'' - xy' + y = 0\ (x > 0)$ is $\phi_1(x) = x$. Find the solution φ of $x^2y'' - xy' + y = x^2$. **7**

b) Find the general solution of $y'' + y = \tan\ x;\ \frac{-\pi}{2} < x < \frac{\pi}{2}$. **7**

4. a) Show that : For any real x_0 and constants α, β there exists a solution ϕ of initial value problem,

$L\ (y) = y'' + a_1y' + a_2y = 0$, $y(x_0) = \alpha$, $y'(x_0) = \beta$ on $-\infty < x < \infty$. **7**

b) One solution of $x^3y''' - 3x^2y'' - 6xy' - 6y = 0$ for x > 0 is $\phi_1(x) = x$. Find the remaining two independent solutions for x > 0. **7**

5. a) Determine whether the functions ϕ_1, ϕ_2 defined on $-\infty < x < \infty$ are LI or LD,

$\phi_1(x) = x$, $\phi_2(x) = |x|$. **5**

b) Show that : For ϕ be any solution of $L(y) = y'' + a_1\ y' + a_2y = 0$ on an interval I-containing a point x_0. Then for all x in I,

$\|\phi(x_0)\|e^{-k|x-x_0|} \leq \|\phi(x)\| \leq \|\phi(x_0)\|.e^{k|x-x_0|}$ Where $\|\phi(x)\| = \left[1\phi(x)|^2 + 1\phi^1(x)|^2\right]^{1/2}$ and $k = 1 + |a_1| + |a_2|$. **9**

6. a) Consider, $L(y) = y'' + a_1(x).y' + a_2(x)y = 0$. Where a_1 and a_2 are uniquely determined by the basis ϕ_1 and ϕ_2. for the solutions of L(y) = 0 and show that,

$$a_1 = \frac{-\begin{vmatrix} \phi_1 & \phi_2 \\ \phi_1'' & \phi_2'' \end{vmatrix}}{W(\phi_1, \phi_2)} \quad a_2 = \frac{\begin{vmatrix} \phi_1' & \phi_2' \\ \phi_1'' & \phi_2'' \end{vmatrix}}{W(\phi_1, \phi_2)}.$$ **7**

b) Find all solutions of equation for $x > 0$

$$x^2y'' - 5xy' + 9y = x^3$$ **7**

7. a) Find first four successive approximations of initial value problem,

$$y' = 3y + 1, \quad y(0) = 2$$ **7**

b) Show that : A function ϕ is a solution of initial value problem

$y' = F(x, y),\ y(x_0) = y_0$ on an interval I iff it is a solution of integral equation,

$y = y_0 + \int_{x_0}^{x} F(t, y)$ at on I. **7**

SLR-VA – 5

Seat No.	

M.Sc. – I (Semester – I) Examination, 2014
MATHEMATICS
Classical Mechanics (Paper – V)

Day and Date : Wednesday, 30-4-2014 Total Marks : 70
Time : 11.00 a.m. to 2.00 p.m.

Instructions *: 1) Q.* ***1*** *and Q.* ***2*** *are* ***compulsory****.*
2) Attempt ***any three*** *questions from Q.* ***3*** *to Q.* ***7****.*
3) Figures to the ***right*** *indicate* ***full*** *marks.*

1. A) Choose the correct alternatives (**1** mark **each**) : **6**

1) A particle of mass M is moving on the surface of a sphere then its degrees of freedom is/are ____________
a) 1 b) 2 c) 3 d) 4

2) An expression which represents holonomic constraints is a function of __________
a) only time b) only position vectors
c) position vectors and time d) constants

3) Shortest distance between two points in a polar plane is a _________
a) parabola b) cycloid c) circle d) straight line

4) Any nontrivial real orthogonal matrix has __________ eigen values.
a) one and only one b) only two
c) only three d) only four

5) Trace of an antisymmetric matrix is _________
a) 0 b) one c) two d) three

6) To obtain Eulerian angles, how many rotations are required in a specific order ?
a) 1 b) 2 c) 3 d) 4

P.T.O.

B) Fill in the blanks : **8**

7) Lagrangian (L) is a function of __________

8) Kinetic energy of a dynamical system is a function of ___________

9) A linear transformation $x_i' = a_{ij} x_j$, i, j = 1, 2, 3 is said to be orthogonal if __________

10) The order of Euler-Lagrange's differential equation of the functional $I = \int_{x_1}^{x_2} F(x, y, y', y'')\, dx$ is ____________

11) A problem which determine extremum of one functional subject to another functional whose value is known called as _______

12) A coordinate q_k is said to be cyclic in Lagrangian L if ___________

13) Lagrangian of a single particle in a plane is defined as ____________

14) A curve with fixed perimeter which encloses maximum area is a ___________

2. a) Define generalized coordinates and derive an expression of generalized velocity. **4**

b) If q_j is cyclic in L then prove that it is cyclic in H. **4**

c) Define Caley-Kleien parameters. **3**

d) State D'Alembert's principle. **3**

3. a) Explain Atwood's machine and find its equation of motion. **6**

b) With usual notations prove that $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right) - \frac{\partial L}{\partial q_j} = Q_j$. **8**

4. a) State and prove fundamental lemma of calculus of variations. **6**

   b) Explain and solve Brachistochrone problem. **8**

5. a) Find the equation of motion of a simple pendulum in Hamiltonian formulation. **7**

   b) Define Routhian and derive Routhian's equations of motion. **7**

6. a) Obtain the matrix of transformation from space axes to body axes in terms of Eulerian angles. **9**

   b) Write a short note on generalized coordinates of a rigid body. **5**

7. a) Find the extremal of the following functional :

   $$J[y(x)] = \int_{x_0}^{x_1} \sqrt{1+y'^2}\, dx, \text{ where } y' = \frac{dy}{dx}.$$ **6**

   b) Prove that K.E. of a dynamical system is a homogeneous quadratic function of generalized velocities. **8**

Seat No.	

M.Sc. (Part – I) (Semester – II) Examination, 2014
MATHEMATICS (Paper – VI)
Algebra – II

Day and Date : Tuesday, 22-4-2014 Max. Marks : 70
Time : 11.00 a.m. to 2.00 p.m.

Instructions : *1) Q. No.* ***1*** *and* ***2*** *are* ***compulsory****.*
2) Attempt ***any three*** *from Q. No.* ***3*** *to* ***7****.*
3) Figures to the ***right*** *indicate* ***full*** *marks.*

1. A) Define the following (**1** mark **each**) :
   1) Simple extension
   2) Separable element
   3) Fixed field
   4) Normal extension.

   B) Fill in the blanks (**1** mark **each**) :
   5) Let K be an extension of F. Then $a \in K$ is algebraic over F iff F(a) is ______________ of F.
   6) Degree of $\sqrt{2}+\sqrt{3}$ over Q is _______________
   7) Let K_H be the fixed field of H. Then $G(K_1 K_H) =$ _______________
   8) A finite, normal and separable extension is called _______________
   9) A field F is called perfect if all finite extensions of F are ____________

   C) State whether the following are **true** or **false** (**1** mark **each**) :
   10) Every finite extension K of F is algebraic.
   11) Splitting field exists for every $f(x) \in F[x]$.
   12) There is a field with 10 elements.
   13) The multiplicative group of non zero elements of a field is cyclic.
   14) A regular pentagon is constructible.

2. a) If a, b $\in$ K are algebraic over F of degrees m and n respectively and if m and n are relatively prime, prove that F(a, b) is of degree mn over F. **4**

   b) Construct a field with 9 elements. **3**

   c) Show that $8x^3 - 6x - 1$ is irreducible over Q. **3**

   d) Show that it is impossible, by straight edge and compass alone, to trisect 60°. **4**

3. a) Let K be an extension of a field F. Show that the elements in K which are algebraic over F form a subfield of K. **7**

   b) If L is an algebraic extension of K and K is an algebraic extension of F, show that L is an algebraic extension of F. **7**

4. a) Show that $Q\left(\sqrt{2}, \sqrt{3}\right) = Q\left(\sqrt{2} + \sqrt{3}\right)$. **7**

   b) Let Q be the field of rational numbers. Determine the degree of the splitting field of the polynomial $x^3 - 2$ over Q. **7**

5. a) Show that a polynomial of degree n over a field can have atmost n roots in any extension field. **7**

   b) Show that the polynomial f(x) $\in$ F[x] has a multiple root if and only if f(x) and $f'(x)$ have a nontrivial common factor. **7**

6. a) Show that any finite extension of a field of characteristic zero is a simple extension. **7**

   b) Show that any field of characteristic zero is perfect. **7**

7. a) Show that the polynomial $x^7 - 10x^5 + 15x + 5$ is not solvable by radicals over Q. **7**

   b) If K is a field and $\sigma_1, \sigma_2, \ldots, \sigma_n$ are distinct automorphisms of K then show that they are linearly independent. **7**

Seat No.	

M.Sc. – I (Semester – II) Examination, 2014
MATHEMATICS
Real Analysis – II (Paper – VII)

Day and Date : Thursday, 24-4-2014 Max. Marks : 70
Time : 11.00 a.m. to 2.00 p.m.

Instructions : *1) Q. No. **1** and **2** are **compulsory**.*
*2) Attempt **any three** from Q. No. **3** to Q. No. **7**.*
*3) Figures to the **right** indicate **full** marks.*

1. A) Fill in the blanks : **5**

   1) If $m^*(A) = 0$, then $m^*(A \cup B) =$ ____________

   2) If A and B be any two sets, then

   $\chi_{A \cup B} = \chi_A + \chi_B -$ ___________

   3) Let f be bounded measurable function on a set E of finite measure, then f is _____________ over E.

   4) If f is monotone on (a, b), then it is ______________ a.e. on (a, b).

   5) If f is Lipschitz on [a, b], then it is ___________ on [a, b].

   B) State whether **true** or **false** : **7**

   1) The canter set C is of measure zero.

   2) Let f be integrable over E. Assume that A and B be two disjoint measurable subsets of E, than

   $$\int_{A \cup B} f = \int_A f + \int_{E-B} f$$

   3) There exists a strictly increasing function on [0, 1] which is continuous only at irrational number in [0, 1].

   4) If f is of bounded variation on [a, b], then f^1 need not be integrable over [a, b].

5) Linear combination of two functions of bounded variation is also of bounded variation.

6) If ϕ is differentiable on (a, b) and ϕ' is increasing, then ϕ is a convex function.

7) The outer measure of an interval is its length.

C) Define the following : **2**

1) Simple function

2) Measurable set.

2. a) Show that a set E is measurable if and only if for each $\varepsilon > 0$, there is a closed set F and open set O for which $F \subseteq E \subseteq O$ and $m^* (0 \sim F) < \varepsilon$. **4**

b) Define $f(x) = \begin{cases} x^2 \cos\left(\frac{1}{x^2}\right) & \text{, if } x \neq 0, x \in [-1, 1] \\ 0 & \text{, if } x = 0 \end{cases}$

Is f of bounded variation on [–1, 1] ? **3**

c) Show that a continuous function on (a, b) is convex if and only if

$\phi\left(\frac{x_1 + x_2}{2}\right) \leq \frac{\phi(x_1) + \phi(x_2)}{2}$, $\forall\, x_1, x_2 \in (a, b)$. **3**

d) Let f be a continuous function on [0, 1] that is absolutely continuous on $[\varepsilon, 1]$, for each $0 < \varepsilon < 1$, show that f may not be absolutely continuous on [0, 1]. **4**

3. a) Let $\{h_n\}$ be a sequence of non-negative integrable functions on E. Suppose $\{h_n(x)\} \to 0$ for almost all $x \in E$. Then show that $\lim_{n \to \infty} \int_E h_n = 0$ if and only if $\{h_n\}$ is uniformly integrable over E. **7**

b) For each of the two functions f on $[1, \infty)$ defined below, show that $\lim_{n \to \infty} \int_1^n f$ exists while f is not integrable over $[1, \infty)$.

i) Define $f(x) = \frac{(-1)^n}{n}$, for $n \leq x < n + 1$

ii) Define $f(x) = \frac{\sin x}{x}$, for $1 \leq x < \infty$. **7**

4. a) Let $\{f_n\}$ be a sequence of measurable functions on E that converges pointwise a.e. on E to f, then prove that f is measurable. **7**

b) Let E be a set having finite measure. Show that $\exists$ an F_σ set F and ε G_δ set G such that $F \subseteq E \subseteq G$ and

$m^*(F) = m^*(E) = m^*(G)$. **7**

5. a) Let f be integrable over E and C be a measurable subset of E, then show that

$$\int_C f = \int_E f \cdot \chi_C .$$ **6**

b) State and prove bounded convergence theorem. Further justify whether this theorem holds for Riemann integral ? **8**

6. a) Suppose f is a real-valued function on $\mathbb{R}$ such that $f^{-1}(c)$ is measurable for each real number C. Is f necessarily measurable ? **7**

b) Show that there is a continuous, strictly increasing function on [0, 1] that maps a set of positive measure onto a set of measure zero. **7**

7. a) Show that if E_1 and E_2 are measurable, then

$m(E_1 \cup E_2) + m(E_1 \cap E_2) = m(E_1) + m(E_2)$. **7**

b) Show that every interval is a Borel set. **7**

Seat No.	

M.Sc. (Part – I) (Semester – II) Examination, 2014
MATHEMATICS (Paper – VIII)
General Topology

Day and Date : Saturday, 26-4-2014 Max. Marks : 70
Time : 11.00 a.m. to 2.00 p.m.

Instructions : *1) Attempt **five** questions.*
*2) Q. No. (**1**) and Q. No. (**2**) are **compulsory**.*
*3) Attempt **any three** from Q. No. (**3**) to Q. No. (**7**).*
*4) Figures to the **right** indicate **full** marks.*

1. A) State whether the following statements are **true** or **false** (**1** mark for **each**) :

1) If τ is the usual topology on R then $\left\{\left(\frac{-1}{n}, \frac{1}{n}\right) : n \in N\right\}$ is a local base at 0.
2) Every metric space is not first countable.
3) Every Lindelof space is a compact space.
4) Regular space did not be T_2-space.
5) Compact Hausdorff space is regular.
6) Regularity of a topological space is not hereditary property.
7) Every normal space is completely normal.
8) If A, B are both open and closed subsets of a topological space X then X is discrete topological space.
9) Arbitrary union of connected sets is connected.
10) If X is a nonempty set then $\mathscr{B} = \{\{x\} | x \in X\}$ is a basis for discrete topological space.

B) Fill in the blanks : **4**

11) Every element of a topology is
12) Set of all limit points of a Set A is called as
13) If (X, τ) is a topological space and $Y \subseteq X$ then topology for Y is called as
14) If $\overline{A} \subseteq A$ then A is

P.T.O.

2. a) Prove that every singleton of a Hausdorff space is closed. **4**

b) If x and y are two distinct points of a Tychonoff space (X, τ) then prove that there exist a real valued continuous function f of X such that $f(x) \neq f(y)$. **4**

c) If X = {a, b, c} and $\tau = \{\phi, X, \{a\}, \{b, c\}\}$ then prove that (X, τ) is a normal space. **3**

d) Prove that any set which contains a neighbourhood of a pont x is also neighbourhood of x. **3**

3. a) Prove that any continuous image of compact set is compact. **7**

b) If (Y, τ^*) is a subspace of a topological space (X, τ) then prove that compact subset in (Y, τ^*) is also compact in (X, τ). **7**

4. a) If A, B are separations of a topological space (X, τ) and E is connected subset of X then prove that either $E \subseteq A$ or $E \subseteq B$. **6**

b) Prove that a subspace of the real line R is connected if and only if it is an interval. **8**

5. a) Prove that every T_1-space is T_0-space. Is the converse true. Explain. **7**

b) Prove that every compact subset E of a Hausdorff space X is closed. **7**

6. a) Prove that every T_4-space is a Tychonov space. **7**

b) Prove that a closed subspace of a normal space is a normal space. **7**

7. a) If A, B are subsets of the topological space (X, τ) and d (A) is a derived set of A then prove that the followings :

i) If $A \subseteq B$ then $d(A) \subseteq d(B)$

ii) $d(A \cup B) = d(A) \cup d(B)$. **7**

b) If (X, τ) is a topological space and $A \subset X$ then prove that A is open if and only if it contains neighbourhood of each of its points. **7**

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Seat No.	

M.Sc. – I (Semester – II) Examination, 2014
MATHEMATICS
Complex Analysis (Paper – IX)

Day and Date : Tuesday, 29-4-2014 Max. Marks : 70

Time : 11.00 a.m. to 2.00 p.m.

Instructions : *1) Q. No. **1** and Q. No. **2** are **compulsory**.*
*2) Solve **any three** questions from Q. No. **3** to Q.No. **7**.*
*3) Figures to the **right** indicate **full** marks.*

1. A) Fill in the blanks : **6**

1) If $z_2, z_3, z_4 \in \mathbb{C}_\infty$ and $S : \mathbb{C}_\infty \to \mathbb{C}_\infty$ is Mobius transformation defined by s (z) = ________ in case $z_4 = \infty$ and $s(z_2) = 1$, $s(z_3) = 0$, $s(z_4) = \infty$.

2) If z_1, z_2, z_3, z_4 be four distinct points in $\mathbb{C}_\infty$. Then $(z_1, z_2, z_3, z_{4)}$ is real number iff _________

3) If f is analytic in the disk B (a ; R) then f is _______

4) If f is analytic in the disk B (a ; R) and suppose that γ is closed rectifiable curve in B (a ; R) then $\int_\gamma^f$ = ________.

5) If γ is a closed rectifiable curve in $\mathbb{C}$ then for $a \notin \{\gamma\}$ index of γ with respect to the point a is given by _____

6) The function $\frac{\sin z}{z}$ has ______ singularity.

P.T.O.

B) Choose correct alternative. **6**

1) Every Mobius transformation have _____ fixed point.
   a) At least one b) At least two
   c) At most one d) At most two

2) Let $\phi : [a, b] \times [c, d] \to$ " be a continuous function. Define

   $$g(t) = \int_a^b \phi(s, t)ds \ \forall \ t \in [c, d] \text{ then}$$

   a) g is continuous b) g is constant
   c) g is not continuous d) g is not constant

3) A series of the form $\sum_{n=-\infty}^{\infty} a_n(z-a)^n$ is called
   a) Simple series b) Convergent series
   c) Divergent series d) Double series

4) If γ is a closed rectifiable curve and n (γ; w) = 0 for $w \in \mathbb{C} - G$ then γ is
   a) Homotopic to zero b) Homogeneous to zero
   c) Homologous to zero d) Holomorphic to zero

5) If γ is closed rectifiable curve in $\mathbb{C}$ and a $\in$ unbounded component of G then
   a) n (γ ; a) is constant b) n (γ ; a) is not constant
   c) n (γ ; a) $\neq 0$ d) none of these

6) $\int_\gamma \frac{e^{iz}}{z^2} dz =$ _______ where γ (t)= re^{it} and $0 \le t \le 2\pi$
   a) 2π b) -2π c) π d) $-\pi$

C) Define term : **2**

1) Bounded variation

2) Removable singularity.

2. a) Show that Mobius map is uniquely determined by its action on any three distinct points. **4**

   b) Prove that a bounded entire function is constant function. **4**

   c) Evaluate the integral $\int_{\gamma} \frac{\sin z}{z^3} dz$ where γ (t) = re^{it} and $0 \leq t \leq 2\pi$. **3**

   d) If f : G $\to$ $\mathbb{C}$ is analytic function and not constant, a $\in$ G and f (a) = 0 then prove that there is R > 0 such that B (a ; R) $\subset$ G and f (z) $\neq$ 0 for 0 < |z – a| $\subset$ R. **3**

3. a) If G is a region and f : G $\to$ $\mathbb{C}$ is an analytic function such that there is a point a in G with |f (a) | $\geq$ |f (z) | $\forall z \in G$ then prove that f is constant. **7**

   b) State and prove Morera's theorem . **7**

4. a) State and prove Cauchy's integral formula of first version. **7**

   b) Find Laurent series expansion of $\frac{1}{z(z-1)(z-2)}$ in ann (0; 0, 1). **7**

5. a) If G be an open set and f : G $\to$ $\mathbb{C}$ be differentiable function then prove that f is analytic function. **8**

   b) Prove that $\int_{0}^{2\pi} \frac{e^{is}}{e^{is} - z} ds = 2\pi$ where |z|<1. **6**

6. a) Evaluate $\int_{0}^{2\pi} \frac{1}{1 + a\sin\theta} d\theta$ (–1 < a < 1). **7**

   b) State and prove Schwar'z lemma. **7**

7. a) If γ :[a, b] $\to$ $\mathbb{C}$ is piecewise smooth then prove that γ is of bounded variation and $v(\gamma) = \int_{a}^{b} | \gamma'(t) | dt$. **8**

   b) State and prove Cauchy's estimate. **6**
